

**Scenario Based Question (BCSC 1010)**

**Assignment-1**

**Set and Relations**

*Q1 For every set S, (i ) ∅ ⊆ S and (ii ) S ⊆ S.*

***Proof:***

Let *S* be a set.

To show that ∅ ⊆ *S*, we must show that ∀*x(x* ∈ ∅ → *x* ∈ *S)* is true. Because the empty set contains no elements, it follows that *x* ∈ ∅ is always false. It follows that the conditional statement *x* ∈ ∅ → *x* ∈ *S* is always true, because its hypothesis is always false and a conditional statement with a false hypothesis is true. Therefore, ∀*x (x* ∈ ∅ → *x* ∈ *S)* is true. This completes the proof of (*i*). Note that this is an example of a vacuous proof. When we wish to emphasize that a set *A* is a subset of a set *B* but that *A* \_= *B,* we write

*A* ⊂ *B* and say that *A* is a **proper subset** of *B.* For *A* ⊂ *B* to be true, it must be the case that *A* ⊆ *B* and there must exist an element *x* of *B* that is not an element of *A*. That is, *A* is a proper subset of *B* if and only if

∀*x (x* ∈ *A* → *x* ∈ *B)* ∧ ∃*x(x* ∈ *B* ∧ *x* \_∈ *A)*is true.

Venn diagrams can be used to illustrate that a set *A* is a subset of a set *B.*

We draw the universal set *U* as a rectangle. Within this rectangle we draw a circle for *B.* Because *A* is a subset of *B,* we draw the circle for *A* within the circle for *B.* This relationship is shown in below Figure 2.



A useful way to show that two sets have the same elements is to show that each set is a subset of the other. In other words, we can show that if A and B are sets with A ⊆ B and B ⊆ A, then A = B. That is, A = B if and only if ∀x (x ∈ A → x ∈ B) and ∀x(x ∈ B → x ∈ A) or equivalently if and only if ∀x(x ∈ A ↔ x ∈ B), which is what it means for the A and B to be equal.

Q2. What are the ordered pairs in the less than or equal to relation, which contains

(a, b) if a ≤ b, on the set {0, 1, 2, 3}?

Q3. What do the statements ∀x∈R (x2 ≥ 0) and ∃x∈Z (x2 = 1) mean?

Q4. What are the truth sets of the predicates P(x), Q(x), and R(x), where the domain

is the set of integers and P(x) is “|x| = 1,” Q(x) is “x2 = 2,” and R(x) is “|x| =

x.”

Q5. Let R1 be the “less than” relation on the set of real numbers and let R2 be the

“greater than” relation on the set of real numbers, that is, R1 = {(x, y) | x < y}

and R2 = {(x, y) | x > y}. What are R1 ∪ R2, R1 ∩ R2, R1 − R2, R2 − R1,

and R1 ⊕ R2?

Q6. Determine whether the relation R on the set of all Web pages is reflexive,

symmetric, antisymmetric, and/or transitive, where (a, b) ∈ R if and only if

a) everyone who has visitedWeb page a has also visited Web page b.

b) there are no common links found on both Web page a andWeb page b.

c) there is at least one common link onWeb page a and Web page b.

d) there is a Web page that includes links to both Web page a andWeb page b.